# Nordic Collegiate Programming Contest NCPC 2021

## October 9, 2021



### Problems

- A Antenna Analysis
- **B** Breaking Bars
- C Customs Controls
- **D** Deceptive Directions
- E Eavesdropper Evasion
- F Fortune From Folly
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- H Hiring Help
- I Intact Intervals
- J Joint Jog Jam
- K Knot Knowledge
- L Locust Locus
- M Marvelous Marathon

Do not open before the contest has started.

#### Advice, hints, and general information

- The problems are **not** sorted by difficulty.
- Your solution programs must read input from *standard input* (e.g. System.in in Java or cin in C++) and write output to *standard output* (e.g. System.out in Java or cout in C++). For further details and examples, please refer to the documentation in the help pages for your favorite language on Kattis.
- For information about which compiler flags and versions are used, please refer to the documentation in the help pages for your favorite language on Kattis.
- Your submissions will be run multiple times, on several different inputs. If your submission is incorrect, the error message you get will be the error exhibited on the first input on which you failed. E.g., if your instance is prone to crash but also incorrect, your submission may be judged as either "Wrong Answer" or "Run Time Error", depending on which is discovered first. The inputs for a problem will always be tested in the same order.
- If you think some problem is ambiguous or underspecified, you may ask the judges for a clarification request through the Kattis system. The most likely response is "No comment, read problem statement", indicating that the answer can be deduced by carefully reading the problem statement or by checking the sample test cases given in the problem, or that the answer to the question is simply irrelevant to solving the problem.
- In general we are lenient with small formatting errors in the output, in particular whitespace errors within reason, and upper/lower case errors are often (but not always) ignored. But not printing any spaces at all (e.g. missing the space in the string "1 2" so that it becomes "12") is typically not accepted. The safest way to get accepted is to follow the output format exactly.
- For problems with floating point output, we only require that your output is correct up to some error tolerance. For example, if the problem requires the output to be within either absolute or relative error of  $10^{-4}$ , this means that
  - If the correct answer is 0.05, any answer between 0.0499 and .0501 will be accepted.
  - If the correct answer is 500, any answer between 499.95 and 500.05 will be accepted.

Any reasonable format for floating point numbers is acceptable. For instance, "17.000000", "0.17e2", and "17" are all acceptable ways of formatting the number 17. For the definition of reasonable, please use your common sense.

### Problem A Antenna Analysis Time limit: 4 seconds

Åke has heard that there may be some suspicious 5G radiation in his city. To test this, he uses the antenna on his roof to measure the 5G level each day. However, he does not know how he should analyze the data.

We are given the measurements for n consecutive days as a list of numbers  $x_1, \ldots, x_n$  (where  $x_i$  denotes the measurement for day i) and a constant c that measures how much Åke expects the radiation to vary from day to day. We want to find, for each day i, the most significant difference between the measurement on day i and any earlier day, after the expected variations are taken into account. More precisely, the goal is to find the maximum value of

$$|x_i - x_j| - c \cdot |i - j|$$

where  $j \leq i$ . I.e., we want to find a large difference in 5G level that has happened recently.

#### Input

The first line of input contains the two integers n and c  $(1 \le n \le 4 \cdot 10^5, 1 \le c \le 10^6)$ , the number of measurements and expected day-to-day variation. The second input line contains the n integers  $x_1, x_2, \ldots, x_n$   $(1 \le x_i \le 10^6 \text{ for } i = 1, 2, \ldots, n)$ , giving the measurements of the n days.

#### Output

Output n integers  $y_1, \ldots, y_n$ , where  $y_i$  is the most significant difference on day i.

| Sample Input 1   | Sample Output 1 |
|------------------|-----------------|
| 5 1<br>2 7 1 5 4 | 0 4 5 3 1       |

### Problem B Breaking Bars Time limit: 2 seconds

Selma is visited by her two grandchildren Elsa and Asle who love chocolate. To be precise, they are especially fond of the brand Nut Cream Puffed Chocolate that comes in bars made up by  $6 \times 6$  squares. The bars can be broken along the valleys between squares into smaller rectangular bars of integer dimensions. Due to the fragile nature of this type of chocolate, the bars often break into smaller rectangular bars even before you unpack them (but still only of integer dimensions).

Thus Selma finds herself with a set of rectangular bars of various dimensions in her candy stash. She knows how important it is to be fair to children, so not only does she want to give Elsa and Asle the same amount of chocolate, but also identical *collections* of rectangular bars (where an  $a \times b$  bar is considered identical to a  $b \times a$  bar). To do this, Selma can break her bars into smaller pieces. A *break* is the operation of taking an  $a \times b$  bar and breaking it along a valley to produce two bars of dimensions  $c \times b$  and  $(a - c) \times b$ , for some integer  $c \in [1, a - 1]$ , or two bars of dimensions  $a \times d$  and  $a \times (b - d)$ , for some integer  $d \in [1, b - 1]$ . See Figure B.1 for an example.

Selma would like to give her two grandchildren identical collections of bars, each collection consisting of at least t squares of chocolate. What is the minimum number of breaks she needs to make to be able to do this?



Figure B.1: Explanation of Sample Input 1. First make a vertical break as shown on the  $3 \times 5$  bar (orange), then make a horizontal break on the newly created  $3 \times 2$  bar (blue). This way Elsa and Asle can each get one  $1 \times 2$ , one  $2 \times 2$ , and one  $3 \times 3$  bar, in total 15 squares each.

#### Input

The first line of input contains two integers n and t  $(1 \le n \le 50, 1 \le t \le 900)$ , where n is the number of bars Selma has, and t is the least number of squares she wants each grandchild to receive. Then follows a line containing n bar descriptions. A bar description is on the format " $a \ge b$ " for two integers  $1 \le a, b \le 6$ .

You may assume that the total amount of chocolate squares among the n bars is at least 2t.

#### Output

Output the minimum number of breaks needed to obtain two identical collections of bars, each having a total of at least t squares.

| Sample Input 1          | Sample Output 1 |
|-------------------------|-----------------|
| 4 15                    | 2               |
| 1x2 2x2 3x3 3x5         |                 |
| Sample Input 2          | Sample Output 2 |
| 6 7                     | 0               |
| 1x2 2x3 1x4 3x2 4x1 6x6 |                 |
| Sample Input 3          | Sample Output 3 |
| 5 3                     | 1               |
| 1x1 1x1 1x1 1x1 1x4     |                 |

### Problem C Customs Controls Time limit: 2 seconds

With lifted restrictions, the border trade between Norway and Sweden will surely be back to its former glory. But the authorities are worried that this will also mean an increase of illegal smuggling of goods. The customs authorities of Norway and Sweden must cooperate to prevent this from becoming too big of a problem.

To pass through the customs, one must visit a series of checkpoints, the Nordic Customs and Passport Control. There are n checkpoints in total, numbered from 1 to n, where 1 is the entrance and n is the exit. There are m pairs of bidirectional roads that connect distinct checkpoints. The *i*th checkpoint takes some amount of time  $t_i$  to pass through, and this is the bottleneck in crossing the border (the time it takes to walk the roads is negligible).

Each checkpoint can be watched by one customs unit, either a Norwegian one or a Swedish one. There are k Norwegian customs units available, and n - k Swedish units. When a road has both of its endpoints watched by customs units from the same country, any smugglers using that road will be caught. Smugglers are of course always in a hurry, and will always attempt to go from 1 to n in as short amount of time as possible.

Your task is to decide where to put the n customs units, so that any smugglers who take a fastest possible route from 1 to n will be caught.

#### Input

The first line of input contains three integers n, m, and k  $(2 \le n \le 10^5, 1 \le m \le 2 \cdot 10^5, 0 \le k \le n)$ , the number of checkpoints, roads, and Norwegian customs units. The second line of input contains n positive integers  $t_1, \ldots, t_n$   $(1 \le t_i \le 10^4)$ , the time it takes to pass through each checkpoint. Then follow m lines of input each containing two integers u and v  $(1 \le u, v \le n)$ , meaning that there is a road between checkpoints u and v.

It is guaranteed that it is possible to go from any checkpoint to any other checkpoint using the roads. There is also at most one road between each pair of checkpoints, and no road connects a checkpoint to itself.

#### Output

If there is a way to place the customs units so that every smuggler is caught, output a string of length n, where the *i*th character indicates which type of customs unit to put at the *i*th checkpoint (an 'N' for a Norwegian customs unit, and an 'S' for a Swedish customs unit). Otherwise, if there is no way to catch every smuggler, output "impossible".

| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|
| 3 2 0          | SSS             |
| 1 1 1          |                 |
| 1 2            |                 |
| 2 3            |                 |

| Sample Input 2 | Sample Output 2 |  |
|----------------|-----------------|--|
| 2 1 1          | impossible      |  |
| 1 1            |                 |  |
| 1 2            |                 |  |
| Sample Input 3 | Sample Output 3 |  |
| 0 0 1          | CNONCONN        |  |

| 8 | 9 | 4 |   |   |   |   |   | SNSNSSNN |
|---|---|---|---|---|---|---|---|----------|
| 3 | 3 | 1 | 2 | 2 | 3 | 2 | 1 |          |
| 1 | 2 |   |   |   |   |   |   |          |
| 1 | 3 |   |   |   |   |   |   |          |
| 1 | 4 |   |   |   |   |   |   |          |
| 2 | 5 |   |   |   |   |   |   |          |
| 3 | 6 |   |   |   |   |   |   |          |
| 4 | 7 |   |   |   |   |   |   |          |
| 5 | 8 |   |   |   |   |   |   |          |
| 6 | 8 |   |   |   |   |   |   |          |
| 7 | 8 |   |   |   |   |   |   |          |

### Problem D Deceptive Directions Time limit: 2 seconds

You find yourself on a remote island, searching for a legendary lost treasure. However, despite having gotten your hands on directions leading straight to the treasure, you have a problem. It turns out you have a saboteur in your expedition, and that at some point they edited the precious directions so they no longer lead to the treasure.

The island can be viewed as a rectangular grid, and the instructions are a sequence of east/west/north/south steps to take in this grid, from a given starting position. These instructions lead straight to the treasure (but may involve walking around obstacles) in the sense that there is no shorter way of reaching the treasure. However, the saboteur has arbitrarily replaced each step of the instructions by a step in one of the other three directions. In other words, any "west" step has been replaced by "east", "north" or "south". This replacement has been done independently for each step, so one "west" may have been replaced by "north" and another by "south", and so on.

Because of this sabotage, the instructions seem pretty useless. But maybe they can still be used to narrow down the search. Write a program to find all possible locations of the treasure.

#### Input

The first line of input consists of two integers w and h ( $3 \le w, h \le 1000$ ), the width and height of the map. Then follow h lines, each containing w characters, describing the map. Each such character is either a '.' symbolizing a walkable space, '#' symbolizing an obstacle such as a body of water, dense forest, or a mountain, or 'S' symbolizing the starting point of the directions.

Finally, there is a line containing a string I ( $1 \le |I| \le 10^5$ ) consisting only of the characters 'NWSE', giving the faulty instruction sequence.

The map has exactly one 'S' and its boundary consists only of obstacle cells. The faulty instruction sequence is such that there is at least one possible location of the treasure.

#### Output

Output the map in the same format as the input (without the first line specifying the dimensions), with all possible locations of the treasure indicated by exclamation marks ('!').

| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|
| 5 5            | ####            |
| ####           | ##              |
| ##             | #!S!#           |
| #.S.#          | #.!.#           |
| ##             | ####            |
| ####           |                 |
| Ν              |                 |

| Sample Input 2                                 | Sample Output 2                           |
|--|---|
| 7 5<br>#######<br>###<br>#S#<br>#*#<br>####### | #######<br>#!.##<br>#S#<br>###<br>####### |
| ESS  |   |

### Problem E Eavesdropper Evasion Time limit: 4 seconds

Alice wants to send n messages to Bob over a communication channel. The *i*th message takes  $t_i$  time steps to send. At each *integer* time step, Alice can start sending any number of her messages. Once started, a message must be transmitted in its entirety (it cannot be paused and resumed later). Any number of messages can be sent in parallel over the channel without affecting the transmission time of individual messages.

An attacker has the capability to disable the security protocols of the channel for an interval of x continuous time steps, but only once (i.e., after doing this, they cannot wait a while and then disable it for another x time steps). While the security is disabled, the attacker is able to listen in, and any message that is sent *in its entirety* during those x time steps is considered exposed.

What is the minimum time needed for Alice to send all *n* messages to Bob so that *at most* two messages are exposed, no matter when the attacker chooses to disable the security?



Figure E.1: Left: Illustration of a solution to Sample Input 1. Right: sending the message of length 4 a time step earlier would not be a solution, because the three messages of length 6, 4, and 3 would then be exposed to an eavesdropper listening in from time step 5 to time step 15.

#### Input

The first line of input contains the two integers n and x ( $1 \le n \le 20000$ ,  $1 \le x \le 10000$ ), the number of messages Alice wants to send and the number of time steps someone may listen in. This is followed by a line containing n integers  $t_1, \ldots, t_n$  ( $1 \le t_i \le 10000$ ), the number of time steps it takes to transmit each message.

#### Output

Output the minimum number of time steps to complete transmission of all n messages so that at most two of them can be exposed.

| Sample Input 1      | Sample Output 1 |
|---------------------|-----------------|
| 6 10<br>2 3 4 5 6 7 | 16              |
|                     |                 |
| Sample Input 2      | Sample Output 2 |
| Sample Input 2      | Sample Output 2 |

### Problem F Fortune From Folly Time limit: 1 second

Your friend Ómar's favourite video game is *Striker-Count*. But he has now grown tired of actually playing the game and is more interested in the lootboxes found in the game. Inside each lootbox there is an item of some level of rarity. Ómar is only interested in acquiring the rarest items in the game. When he starts the game, he chooses two numbers n and k, such that  $k \leq n$ . He then opens lootboxes in the game until k of the last n lootboxes included an item of the highest rarity.

This activity amuses Ómar, but does not interest you in the slightest. You are more interested in the numbers: you know that each lootbox Ómar opens has probability p of containing an item of highest rarity, independently for each lootbox. You want to find the expected number of lootboxes Ómar will open before concluding his process.

#### Input

The only line of the input contains the two integers n and k ( $1 \le k \le n \le 6$ ), and the real number p (0 and <math>p has at most four decimals after the decimal point), with meanings as described above.

#### Output

Output the expected number of lootboxes Ómar must open, with a relative error of at most  $10^{-6}$ . It is guaranteed that the input is such that this expected number does not exceed  $10^9$ .

| Sample Input 1 | Sample Output 1    |  |  |
|----------------|--------------------|--|--|
| 3 2 0.0026     | 74445.39143490087  |  |  |
| Sample Input 2 | Sample Output 2    |  |  |
| 6 1 0.0026     | 384.61538461538464 |  |  |

### Problem G Grazed Grains Time limit: 3 seconds

This year, there have been unusually many UFO sightings reported. Nobody knows if they are caused by optical illusions, weather phenomena, or secret technology being tested by foreign powers (terrestrial or not). UFO enthusiasts across the world rejoice, and speculations run wild. But someone who is not impressed is the farmer Celeste. Her large grain field has repeatedly been used as a landing site by the UFOs, destroying her crops. Celeste would like to bring whoever is responsible to justice, but before she can do so she must assess the damage caused by the unidentified flying offenders.

In total *n* circular UFOs have landed in Celeste's field. The *i*th one left a crop circle which destroyed all crops within radius  $r_i$  of the point  $x_i, y_i$ . The field is very large, so you can assume that it extends infinitely in all directions. Your task is to estimate the total area of crops destroyed by the UFOs. Celeste only needs a rough estimate of the true answer, and your answer will be counted as correct if its relative error is less than 10%.

#### Input

The first line of input contains one integer n  $(1 \le n \le 10)$ , the number of UFOs. Then follow n lines, the *i*th of which contains the three integers  $x_i$ ,  $y_i$ , and  $r_i$   $(0 \le x_i, y_i \le 10, 1 \le r_i \le 10)$ , describing the crop circle left by the *i*th UFO.

#### Output

Output the total area covered by the crop circles in the input, with a relative error of at most 1/10.

| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|
| 1              | 3.1415926535    |
| 0 0 1          |                 |
| Sample Input 2 | Sample Output 2 |
| 4              | 25.991148       |
| 0 0 2          |                 |
| 1 0 1          |                 |
| 222            |                 |
| 10 10 1        |                 |

### Problem H Hiring Help Time limit: 4 seconds

A certain large unnamed software development company has n developers. The productivity of each coder working for the company has been rigorously measured in terms of two key performance indicators: the number of lines of code they write per hour, and the number of bugs they fix per hour.

When a project needs to be done, the manager in charge of the project is allocated some budget of t man-hours of programmer time. The manager can then staff different coders on the project, up to a total of t hours. For instance if there are three programmers, the manager can allocate any non-negative real numbers  $t_1$ ,  $t_2$ , and  $t_3$  hours of their respective work hours, as long as  $t_1 + t_2 + t_3 \le t$ . If the three programmers write  $l_1$ ,  $l_2$ , and  $l_3$  lines of code per hour, a total amount of  $t_1 \cdot l_1 + t_2 \cdot l_2 + t_3 \cdot l_3$  lines of code will then be written for the project. Similarly if they fix  $b_1$ ,  $b_2$ , and  $b_3$  bugs per hour, a total of  $t_1 \cdot b_1 + t_2 \cdot b_2 + t_3 \cdot b_3$  bugs will be fixed.

Due to the uncertain economy, the company has a hiring freeze, meaning that no new coders are hired to the company. However, under certain conditions, a manager is allowed to bring in outside help by outsourcing a project to an external consultant rather than doing it in-house. But this is only allowed if it is not possible to do the project equally efficiently in-house. In particular, if the consultant writes  $\ell$  lines of code and fixes b bugs in t hours, and there exists some allocation of our existing coders which would write at least  $\ell$  lines of code and fix at least b bugs in at most t hours, then a manager is not allowed to hire this consultant (regardless of whether those existing coders would actually have time to work on the project or whether they are already too busy with other projects).

While no new coders are hired, employees do sometimes decide to leave the company. Given a chronological list of events – requests to use a consultant, and employees quitting – find out which of the requests will be approved.

#### Input

The first line of input consists of a single integer n ( $0 \le n \le 2 \cdot 10^5$ ), the number of coders (initially) at the company. The employees are numbered from 1 to n (names are too personal). Then follow n lines, the *i*th of which contains two integers  $\ell_i$  and  $f_i$  ( $1 \le \ell_i, f_i \le 10^8$ ), the number of lines of code and the number of bugs fixed per hour by coder i.

Next follows a line with a single integer e ( $1 \le e \le 10^5$ ), the number of events. This is followed by e lines, describing the events in chronological order. An event is a line in one of the following two forms:

- "c  $t \ \ell f$ ", for three integers  $t, \ell$  and  $f \ (1 \le t \le 100, 1 \le \ell, f \le 10^8)$ : a request to take in a consultant for a project of t hours, where the consultant would write  $\ell$  lines of code and fix f bugs in those t hours.
- "q i", for an integer  $i (1 \le i \le n)$ : coder i quit the company.

You may assume that no coder quits more than once.

#### Output

For each request to take in a consultant, output "yes" if the request is approved, and "no" if it is not approved.

| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|
| 4              | no              |
| 200 100        | no              |
| 100 200        | yes             |
| 100 100        | no              |
| 200 200        |                 |
| 5              |                 |
| c 10 2000 2000 |                 |
| c 5 750 750    |                 |
| q 4            |                 |
| c 3 600 600    |                 |
| c 10 1500 1500 |                 |

| Sample Input 2 | Sample Output 2 |
|----------------|-----------------|
| 8              | no              |
| 400 300        | no              |
| 300 200        | no              |
| 300 400        | no              |
| 200 300        | yes             |
| 500 500        | no              |
| 100 500        | no              |
| 100 100        | no              |
| 500 100        |                 |
| 12             |                 |
| c 4 1611 1601  |                 |
| c 3 602 601    |                 |
| c 2 399 795    |                 |
| c 1 395 206    |                 |
| q 7            |                 |
| q 6            |                 |
| q 5            |                 |
| q 4            |                 |
| c 4 1611 1601  |                 |
| c 3 602 601    |                 |
| c 2 399 795    |                 |
| c 1 395 206    |                 |

### Problem I Intact Intervals Time limit: 5 seconds

Gustav is an astronaut on the Nordic Celestial Planetary Craft (NCPC), a large space station in orbit around Mars. Today, one of Gustav's tasks is to look over the satefy routines on board.

The space station consists of n modules arranged in a circle, so that module i is connected to module i + 1 for  $i = 1 \dots n - 1$ , and module n is connected to module 1. Each module i has a non-negative integer type  $a_i$ , representing the kind of equipment that can be found there. Different modules can have the same type. In case of emergency, the equipment must be rearranged so that each module i instead gets type  $b_i$ , for some list  $b_1, b_2, \dots, b_n$ . Here, the list bis a rearrangement of the list a.

Gustav has noticed that if some module connections are severed, causing the space station to split into separate parts, it may become impossible to perform this rearrangement of the equipment. He decides to estimate how likely it is that the safety routines can be followed, by calculating in how many ways the space station can be separated into two or more parts such that it is still possible to rearrange the equipment according to the emergency procedures.

In other words, your task is to count in how many ways the circular list a can be partitioned into *at least two* non-empty contiguous intervals, in such a way that the circular list b can be obtained by rearranging elements within each interval. Since this number can be quite big, you should find its remainder modulo  $10^9 + 7$ .

For example, consider Sample Input 1 below. Here the list a could be split into [1|223|4], indicating that the connection between modules 1 and 2, and the connection between modules 4 and 5, are severed. Note that the connection between module 5 and 1 remains in this split. The second possible way in which a could be split is [12|2|34].

In Sample Input 2 below, the only possible way to split the list a into at least two non-empty parts is to separate the two modules. But then it is impossible to rearrange the parts to create the list b. Hence, the answer is 0.

#### Input

The first line of input contains a single integer n ( $2 \le n \le 10^6$ ), the number of modules. The second line contains the n integers  $a_1, \ldots a_n$  ( $0 \le a_i \le 10^9$ ). The third and final line contains the n integers  $b_1, \ldots, b_n$  ( $0 \le b_i \le 10^9$ ).

The list b is guaranteed to be a rearrangement of the list a.

#### Output

Print one integer, the number of safe separations modulo  $10^9 + 7$ .

| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|
| 5              | 2               |
| 1 2 2 3 4      |                 |
| 4 3 2 2 1      |                 |

| Sample Input 2 | Sample Output 2 |  |  |  |  |  |
|----------------|-----------------|--|--|--|--|--|
| 2              | 0               |  |  |  |  |  |
| 1 2            |                 |  |  |  |  |  |
| 2 1            |                 |  |  |  |  |  |

### Problem J Joint Jog Jam Time limit: 1 second

Like so many good stories, this one begins with a claim that Kari is a faster runner than Ola, who of course challenges Kari to a run-down. Dubbed (rather ironically) Non-Competitive Pace Challenge, they want to see who can run the furthest in a certain amount of time t. Clearly they both choose to run in straight lines with constant speed.

Kari wrote an app to make sure that Ola does not cheat, but the app requires that their phones constantly communicate over Bluetooth.

After their run, Kari needs to ensure that they were never too far apart from each other at any time during the run. Write a program that computes the maximum distance between Kari and Ola at any point during the run.

#### Input

The input consists of a single line containing eight integers describing four points:

- the starting position of Kari,
- the starting position of Ola,
- the ending position of Kari, and
- the ending position of Ola,

in that order. Each point is given by two integers x and y ( $0 \le x, y \le 10^4$ ), the coordinates of the point.

#### Output

Output the maximum distance between Kari and Ola during their run, with an absolute or relative error of at most  $10^{-6}$ .

| Sample Input 1    | Sample Output 1 |
|-------------------|-----------------|
| 0 0 0 0 1 1 2 2   | 1.4142135624    |
| Sample Input 2    | Sample Output 2 |
| 0 0 0 1 0 2 2 1   | 2.2360679775    |
| Sample Input 3    | Sample Output 3 |
| 5 0 10 0 5 0 10 0 | 5               |

### Problem K Knot Knowledge Time limit: 1 second

Sonja the scout is taking a test to see if she knows all the knots a scout is supposed to know. The Scout's Big Book of Knots has descriptions of  $1\,000$  different knots, conveniently numbered from 1 to  $1\,000$ . For the test, Sonja needs to learn a specific set of n of these knots. After some intense studying, she has learned all except one of them, but she has forgotten which knot she does not yet know.

Given the list of knots Sonja needs to learn, and the ones she has learned so far, find the remaining knot to learn.

#### Input

The first line of input consists of an integer n  $(2 \le n \le 50)$ , the number of knots Sonja needs to learn. This is followed by a line containing n distinct integers  $x_1, \ldots, x_n$   $(1 \le x_i \le 1\,000)$ , the knots that Sonja needs to learn. Finally, the last line contains n - 1 distinct integers  $y_1, \ldots, y_{n-1}$   $(1 \le y_i \le 1\,000)$ , the knots that Sonja has learned so far. You may assume that each knot Sonja has learned is one of the n knots she was supposed to learn.

#### Output

Output the number of the remaining knot that Sonja needs to learn.

| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|
| 4              | 1               |
| 1 2 4 3        |                 |
| 4 2 3          |                 |
|                |                 |

| Sample Input 2 | Sample Output 2 |
|----------------|-----------------|
| 4              | 10              |
| 10 101 999 1   |                 |
| 1 999 101      |                 |

### Problem L Locust Locus Time limit: 1 second

There are two different species of periodical cicadas that only appear from hibernation every 13 and 17 years, respectively. Your old grandpa tells you that he saw them simultaneously back in '92. You start pondering how many years you have to wait until you see them again. You collect information about other pairs of periodical cicadas and when they were last observed to find out when the next simultaneous appearance is.

Given several different pairs of cicadas and their last simultaneous appearance, find the next year that one of the pairs reappears.

#### Input

The first line of input contains a single integer k ( $1 \le k \le 99$ ), the number of pairs of periodical cicadas. Then follow k lines, each containing three integers y,  $c_1$  and  $c_2$  ( $1800 \le y \le 2021$ ,  $1 \le c_1, c_2 \le 99$ ), the year this pair was last observed and cycle lengths for the first and second species, respectively. You may assume that none of the k pairs reappears earlier than 2022.

#### Output

Output the first year a pair reappears.

| Sample Input 1 | Sample Output 1 |
|----------------|-----------------|
| 3              | 2036            |
| 1992 13 17     |                 |
| 1992 14 18     |                 |
| 2001 5 7       |                 |
| Sample Input 2 | Sample Output 2 |

| · · ·    | • •  |
|----------|------|
| 2        | 2026 |
| 2020 2 3 |      |
| 2019 3 4 |      |

### Problem M Marvelous Marathon Time limit: 5 seconds

A marathon race is being planned in the beautiful countryside. The course will be somewhere along a long, bidirectional, road. The organizers want to determine exactly where along this road the race should be in order to maximize the experience for the runners, so that they get to enjoy as much beautiful scenery as possible and are distracted from their tired limbs. The scenery varies in beauty depending on where on the road you are, and also in which direction you are running. Because of this, the organizers are fine with having the runners make *up to* two U-turns, as long as no part of the road is used more than once in each direction.

We model the road of length m meters as a rectangular grid of size  $2 \times m$ , where each cell has a non-negative "beauty" value associated with it. The columns represent each meter of the road ordered from start to to end. The top row in a column represents the beauty for this part of the road when running in the direction towards the end of the road, and the bottom row in a column represents the beauty when running towards the start of the road. A race of length x is then some set of exactly x of the cells in the grid. Those x cells must form a path in the grid where no cell is visited more than once, we only move to the right or down from cells in the top row, and we only move to the left or up from cells in the bottom row. See Figure M.1 for an example race. The "total beauty" of a race is the sum of the beauty values of the included cells.

The road is long, so rather than providing a list of all of the 2m beauty values, each side of the road is divided into a small number of segments, where the cells within a segment have some constant beauty value (and cells with beauty 0 are simply omitted).

| 9 | 9 | 9 | 4 | 4 | 4- | ▶4  |     |     |     |     | 6-          | ►6- | ►6 | 6 |   |   |   |
|---|---|---|---|---|----|-----|-----|-----|-----|-----|-------------|-----|----|---|---|---|---|
|   |   |   |   |   | 74 | -7◄ | -7◄ | -7- | -7◄ | -7◄ | <b>-</b> 7◄ | -74 | -7 |   | 5 | 8 | 8 |

Figure M.1: Illustration of Sample Input 1. The numbers in the cells indicate the beauty value for each meter of the road (with omitted values being 0). The highlighted cells and arrows mark the optimal race, involving two U-turns.

#### Input

The first line of input contains the three integers m, x and n  $(1 \le m \le 10^9, 1 \le x \le 2m, 0 \le n \le 200)$ , the length of the road, the length of the race and the number of segments.

This is followed by *n* lines describing the segments. Each such line contains three integer a, b, v ( $0 \le a, b \le m, 1 \le v \le 10^9$ , and  $a \ne b$ ), describing a segment with endpoints *a* and *b* having beauty value *v*. If a < b, this is the segments of cells in the top row of the grid in the range [a, b), and if a > b, this is the segments of cells in the bottom row of the grid in the range [b, a).

The parts of the road that are not covered by any segments have beauty value 0. Each cell in the grid is covered at most once (that is, there are no overlapping segments in the same direction).

#### Output

Output the maximum possible total beauty the race can have.

| Sample Input 1       | Sample Output 1 |
|----------------------|-----------------|
| 19 14 6              | 89              |
| 14 5 7               |                 |
| 11 15 6              |                 |
| 374                  |                 |
| 16 15 5              |                 |
| 19 17 8              |                 |
| 0 3 9                |                 |
|                      |                 |
| Sample Input 2       | Sample Output 2 |
| 100000 42195 2       | 3554850000000   |
| 30000 60000 50000000 |                 |

40000 10000 100000000